



## Chapter 1

### Dimensions, Units, and Their Conversion

#### 1.1 Units and Dimensions

**Dimensions** are our basic concepts of measurement such as length, time, mass, temperature, and so on; **units** are the means of expressing the dimensions, such as feet or centimeters for length, and hours or seconds for time.

In this lectures you will use the two most commonly used systems of units:

1. **SI**, formally called Le Systeme Internationale d'Unites, and informally called SI or more often (redundantly) the SI system of units.
2. **AE**, or American Engineering system of units.

Dimensions and their respective units are classified as fundamental or derived:

- **Fundamental** (or basic) dimensions/units are those that can be measured independently and are sufficient to describe essential physical quantities.
- **Derived** dimensions/units are those that can be developed in terms of the fundamental dimensions/units.

Tables 1.1 and 1.2 list both basic, derived, and alternative units in the SI and AE systems. Figure 1.1 illustrates the relation between the basic dimensions and some of the derived dimensions.

One of the best features of the SI system is that (except for time) units and their multiples and submultiples are related by standard factors designated by the **prefix** indicated in Table 1.3.

#### 1.2 Operations with Units

The rules for handling units are essentially quite simple:

##### 1.2.1 Addition, Subtraction, Equality

You can add, subtract, or equate numerical quantities only if the associated units of the quantities are the same. Thus, the operation

$$5 \text{ kilograms} + 3 \text{ joules}$$

cannot be carried out because the units as well as the dimensions of the two terms are different. The numerical operation

$$10 \text{ pounds} + 5 \text{ grams}$$

can be performed (because the dimensions are the same, mass) only after the units are transformed to be the same, either pounds, grams, or ounces, or some other mass unit.



Table 1.1 SI Units

Physical Quantity	Name of Unit	Symbol for Unit*	Definition of Unit
<i>Basic SI Units</i>			
Length	metre, meter	m	
Mass	kilogramme, kilogram	kg	
Time	second	s	
Temperature	kelvin	K	
Molar amount	mole	mol	
<i>Derived SI Units</i>			
Energy	joule	J	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \rightarrow \text{Pa} \cdot \text{m}^3$
Force	newton	N	$\text{kg} \cdot \text{m} \cdot \text{s}^{-2} \rightarrow \text{J} \cdot \text{m}^{-1}$
Power	watt	W	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \rightarrow \text{J} \cdot \text{s}^{-1}$
Density	kilogram per cubic meter		$\text{kg} \cdot \text{m}^{-3}$
Velocity	meter per second		$\text{m} \cdot \text{s}^{-1}$
Acceleration	meter per second squared		$\text{m} \cdot \text{s}^{-2}$
Pressure	newton per square meter, pascal		$\text{N} \cdot \text{m}^{-2}$ , Pa
Heat capacity	joule per (kilogram · kelvin)		$\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
<i>Alternative Units</i>			
Time	minute, hour, day, year	min, h, d, y	
Temperature	degree Celsius	°C	
Volume	litre, liter ( $\text{dm}^3$ )	L	
Mass	tonne, ton (Mg), gram	t, g	

Table 1.2 American Engineering (AE) System Units

Physical Quantity	Name of Unit	Symbol
<i>Some Basic Units</i>		
Length	foot	ft
Mass	pound (mass)	$\text{lb}_m$
Time	second, minute, hour, day	s, min, h (hr), day
Temperature	degree Rankine or degree Fahrenheit	°R or °F
Molar amount	pound mole	$\text{lb mol}$
<i>Derived Units</i>		
Force	pound (force)	$\text{lb}_f$
Energy	British thermal unit, foot pound (force)	Btu, (ft)( $\text{lb}_f$ )
Power	horsepower	hp
Density	pound (mass) per cubic foot	$\text{lb}_m/\text{ft}^3$
Velocity	feet per second	ft/s
Acceleration	feet per second squared	$\text{ft}/\text{s}^2$
Pressure	pound (force) per square inch	$\text{lb}_f/\text{in}^2$ , psi
Heat capacity	Btu per pound (mass) per degree F	$\text{Btu}/(\text{lb}_m)(^\circ\text{F})$



Figure 1.1 Relation between the basic dimensions (in boxes) and various derived dimensions (in ellipses).

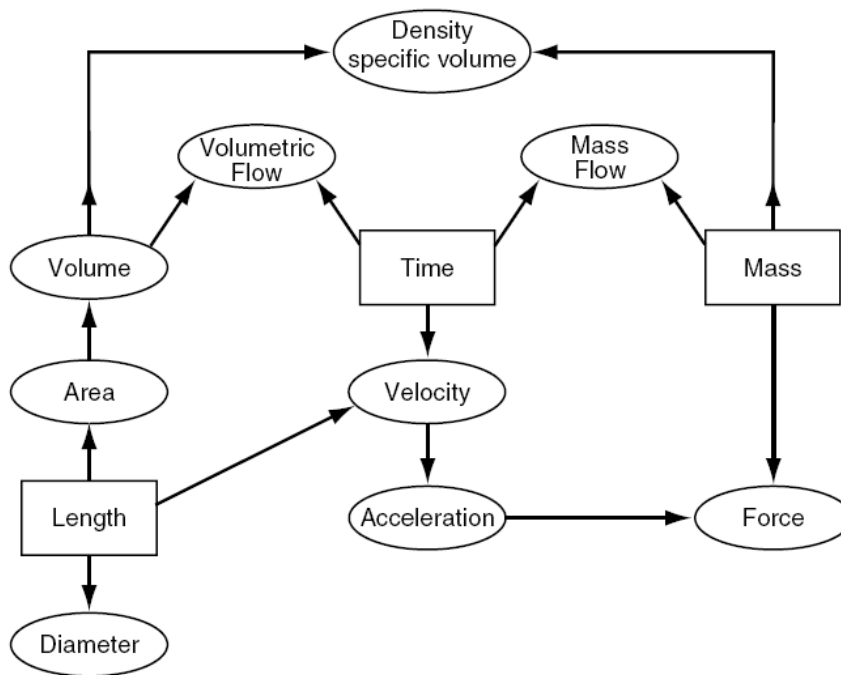


Table 1.3 SI Prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
$10^9$	giga	G	$10^{-1}$	deci	d
$10^6$	mega	M	$10^{-2}$	centi	c
$10^3$	kilo	k	$10^{-3}$	milli	m
$10^2$	hecto	h	$10^{-6}$	micro	$\mu$
$10^1$	deka	da	$10^{-9}$	nano	n

### 1.2.2 Multiplication and Division

You can multiply or divide unlike units at will such as

$$50(\text{kg})(\text{m})/(\text{s})$$

but you cannot cancel or merge units unless they are identical. Thus,  $3 \text{ m}^2/60 \text{ cm}$  can be converted to  $3 \text{ m}^2/0.6 \text{ m}$ , and then to  $5 \text{ m}$ , but in  $\text{m/s}^2$ , the units cannot be cancelled or combined.

#### Example 1.1

Add the following:

- (a) 1 foot + 3 seconds      (b) 1 horsepower + 300 watts

**Solution**



The operation indicated by

$$1 \text{ ft} + 3 \text{ s}$$

has no meaning since the dimensions of the two terms are not the same. In the case of

$$1 \text{ hp} + 300 \text{ watts}$$

the dimensions are the same (energy per unit time), but the units are different. You must transform the two quantities into like units, such as horsepower or watts, before the addition can be carried out. Since  $1 \text{ hp} = 746 \text{ watts}$ ,

$$746 \text{ watts} + 300 \text{ watts} = 1046 \text{ watts}$$

### 1.3 Conversion of Units and Conversion Factors

The procedure for converting one set of units to another is simply to multiply any number and its associated units by ratios termed **conversion factors** to arrive at the desired answer and its associated units.

If a plane travels at twice the speed of sound (assume that the speed of sound is  $1100 \text{ ft/s}$ ), how fast is it going in miles per hour?

We formulate the conversion as follows

$$\frac{2 \times 1100 \text{ ft}}{\text{s}} \left| \frac{1 \text{ mi}}{5280 \text{ ft}} \right| \frac{60 \text{ s}}{1 \text{ min}} \left| \frac{60 \text{ min}}{1 \text{ hr}} \right|$$

$\frac{\text{ft}}{\text{s}} \quad \frac{\text{mi}}{\text{s}} \quad \frac{\text{mi}}{\text{min}}$

#### Example 1.2

- (a) Convert  $2 \text{ km}$  to miles.      (b) Convert  $400 \text{ in.}^3/\text{day}$  to  $\text{cm}^3/\text{min}$ .

#### **Solution**

- (a) One way to carry out the conversion is to look up a direct conversion factor, namely  $1.61 \text{ km} = 1 \text{ mile}$ :

$$\frac{2 \text{ km}}{1} \left| \frac{1 \text{ mile}}{1.61 \text{ km}} \right| = 1.24 \text{ mile}$$

Another way is to use conversion factors you know

$$\frac{2 \text{ km}}{1} \left| \frac{10^5 \text{ cm}}{1 \text{ km}} \right| \left| \frac{1 \text{ in.}}{2.54 \text{ cm}} \right| \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| \left| \frac{1 \text{ mile}}{5280 \text{ ft.}} \right| = 1.24 \text{ mile}$$

$$(b) \frac{400 \text{ in.}^3}{\text{day}} \left| \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right)^3 \right| \left| \frac{1 \text{ day}}{24 \text{ hr}} \right| \left| \frac{1 \text{ hr}}{60 \text{ min}} \right| = 4.55 \frac{\text{cm}^3}{\text{min}}$$

In part (b) note that not only are the numbers in the conversion of inches to centimeters raised to a power, but the units also are raised to the same power.



### Example 1.3

An example of a semiconductor is ZnS with a particle diameter of 1.8 nanometers. Convert this value to (a) dm (decimeters) and (b) inches.

#### Solution

$$(a) \frac{1.8 \text{ nm}}{1} \left| \frac{10^{-9} \text{ m}}{1 \text{ nm}} \right| \left| \frac{10 \text{ dm}}{1 \text{ m}} \right| = 1.8 \times 10^{-8} \text{ dm}$$

$$(b) \frac{1.8 \text{ nm}}{1} \left| \frac{10^{-9} \text{ m}}{1 \text{ nm}} \right| \left| \frac{39.37 \text{ in.}}{1 \text{ m}} \right| = 7.09 \times 10^{-8} \text{ in.}$$

In the AE system the conversion of terms involving pound **mass** and pound **force** deserve special attention. Let us start the discussion with Newton's Law:

$$F = Cma \quad (1.1)$$

Where:

F = force

C = a constant whose numerical value and units depend on those selected for F, m, and a

m = mass

a = acceleration

In the SI system in which the unit of force is defined to be the Newton (N) when 1 kg is accelerated at  $1 \text{ m/s}^2$ , a conversion factor  $C = 1 \text{ N}/(\text{Kg})(\text{m})/\text{s}^2$  must be introduced to have the force be 1 N:

$$F = \frac{1 \text{ N}}{\frac{(\text{kg})(\text{m})}{\text{s}^2}} \left| \frac{1 \text{ kg}}{\tilde{m}} \right| \left| \frac{1 \text{ m}}{\tilde{a}} \right| = 1 \text{ N} \quad (1.1)$$

Because the numerical value associated with the conversion factor is **1**, the conversion factor seems simple, even nonexistent, and the units are ordinarily ignored.

In the **AE** system an analogous conversion factor is required. If a mass of 1 lb<sub>m</sub> is hypothetically accelerated at  $g \text{ ft/s}^2$ , where  $g$  is the acceleration that would be caused by gravity (about  $32.2 \text{ ft/s}^2$  depending on the location of the mass), we can make the force be 1 lb<sub>f</sub> by choosing the proper numerical value and units for the conversion factor C:



$$F = \left( \frac{1(\text{lb}_f)(s^2)}{32.174(\text{lb}_m)(\text{ft})} \right) \left( \frac{1 \text{ lb}_m}{\tilde{m}} \left| \frac{g \text{ ft}}{\tilde{g} s^2} \right| \right) = 1 \text{ lb}_f \quad (1.2)$$

The inverse of the conversion factor with the numerical value **32.174** included is given the special symbol  $g_c$

$$g_c = 32.174 \frac{(\text{ft})(\text{lb}_m)}{(s^2)(\text{lb}_f)}$$

But never forget that the pound (**mass**) and pound (**force**) are not the same units in the **AE** system.

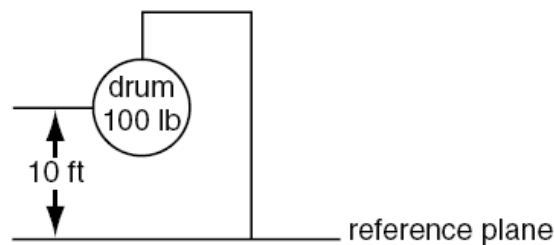
### Example 1.4

What is the potential energy in (ft)(lb<sub>f</sub>) of a 100 lb drum hanging 10 ft above the surface of the earth with reference to the surface of the earth?

### **Solution**

Potential energy =  $P = m g h$

Assume that the 100 lb means 100 lb mass;  $g$  = acceleration of gravity =  $32.2 \text{ ft/s}^2$ . Figure E1.4 is a sketch of the system.



**Figure E1.4**

$$P = \frac{100 \text{ lb}_m}{\tilde{m}} \left| \frac{32.2 \text{ ft}}{s^2} \right| \left| \frac{10 \text{ ft}}{\tilde{g}} \right| \frac{(s^2)(\text{lb}_f)}{32.174(\text{ft})(\text{lb}_m)} = 1000 (\text{ft})(\text{lb}_f)$$

Notice that in the ratio of  $32.2 \text{ ft/s}^2$  divided by  $32.174[(\text{ft})(\text{lb}_m)]/[(s^2)(\text{lb}_f)]$ , the numerical values are almost equal. Many engineers would solve the problem by saying that  $100 \text{ lb} \times 10 \text{ ft} = 1000 (\text{ft})(\text{lb})$  without realizing that, in effect, they are canceling out the numbers in the  $g/g_c$  ratio, and that the lb in the solution means lb<sub>f</sub>.



### Example 1.5

In biological systems, production rate of glucose is  $0.6 \mu\text{g mol}/(\text{mL})(\text{min})$ . Determine the production rate of glucose for this system in the units of  $\text{lb mol}/(\text{ft}^3)(\text{day})$ .

### Solution

Basis: 1 min

$$\begin{aligned} & \frac{0.6 \mu\text{g mol}}{(\text{mL})(\text{min})} \left| \frac{1 \text{ g mol}}{10^6 \mu\text{g mol}} \right| \left| \frac{1 \text{ lb mol}}{454 \text{ g mol}} \right| \left| \frac{1000 \text{ mL}}{1 \text{ L}} \right| \left| \frac{1 \text{ L}}{3.531 \times 10^{-2} \text{ ft}^3} \right| \left| \frac{60 \text{ min}}{\text{hr}} \right| \left| \frac{24 \text{ hr}}{\text{day}} \right| \\ &= 0.0539 \frac{\text{lb mol}}{(\text{ft}^3)(\text{day})} \end{aligned}$$

## 1.4 Dimensional Consistency (Homogeneity)

The concept of dimensional consistency can be illustrated by an equation that represents the pressure/volume/temperature behavior of a gas, and is known as van der Waals's equation.

$$\left( p + \frac{a}{V^2} \right) (V - b) = RT$$

Inspection of the equation shows that the constant **a** must have the units of  $[(\text{pressure})(\text{volume})^2]$  for the expression in the first set of parentheses to be consistent throughout. If the units of pressure are **atm** and those of volume are **cm<sup>3</sup>**, **a** will have the units of  $[(\text{atm})(\text{cm})^6]$ . Similarly, **b** must have the same units as **V**, or in this particular case the units of **cm<sup>3</sup>**.

### Example 1.6

Your handbook shows that microchip etching roughly follows the relation

$$d = 16.2 - 16.2e^{-0.021t} \quad t < 200$$

where **d** is the depth of the etch in microns (micrometers,  $\mu\text{m}$ ) and **t** is the time of the etch in seconds. What are the units associated with the numbers 16.2 and 0.021? Convert the relation so that **d** becomes expressed in inches and **t** can be used in minutes.

### Solution

Both values of **16.2** must have the associated units of microns ( $\mu\text{m}$ ). The **exponential** must be **dimensionless** so that **0.021** must have the associated units of  $\text{s}^{-1}$ .



$$d_{\text{in}} = \frac{16.2 \mu\text{m}}{10^6 \mu\text{m}} \left| \frac{1 \text{ m}}{10^6 \mu\text{m}} \right| \left| \frac{39.27 \text{ in.}}{1 \text{ m}} \right| \left[ 1 - \exp \frac{-0.021}{s} \left| \frac{60s}{1 \text{ min}} \right| \frac{t_{\text{min}}}{1 \text{ min}} \right]$$

$$= 6.38 \times 10^{-4} (1 - e^{-1.26t_{\text{min}}}) \text{ inches}$$

### Nondimensional Groups:

As you proceed with the study of chemical engineering, you will find that groups of symbols may be put together, either by theory or based on experiment, that have no net units. Such collections of variables or parameters are called **dimensionless** or **nondimensional groups**. One example is the Reynolds number (group) arising in fluid mechanics.

$$\text{Reynolds number} = \frac{Dv\rho}{\mu} = N_{RE}$$

where  $D$  is the pipe diameter, say in cm;  $v$  is the fluid velocity, say in cm/s;  $\rho$  is the fluid density, say in g/cm<sup>3</sup>; and  $\mu$  is the viscosity, say in centipoise, units that can be converted to g/(cm)(s). Introducing the consistent set of units for  $D$ ,  $v$ ,  $\rho$ , and  $\mu$  into  $Dv\rho/\mu$ , you will find that all the units cancel out so that the numerical value of 1 is the result of the cancellation of the units.

$$\frac{\text{cm}}{dx} \left| \frac{\text{cm}}{s} \right| \left| \frac{\text{g}}{\text{cm}^3} \right| \left| \frac{(\text{cm})(s)}{\text{g}} \right|$$

### Example 1.7

Explain without differentiating why the following differentiation cannot be correct:

$$\frac{d}{dx} \sqrt{1 + (x^2/a^2)} = \frac{2ax}{\sqrt{1 + (x^2/a^2)}}$$

where  $x$  is length and  $a$  is a constant.

### **Solution**

- Observe that  $x$  and  $a$  must have the same units because the ratio  $x^2/a^2$  must be dimensionless (because 1 is dimensionless).
- Thus, the left-hand side of the equation has units of  $1/x$  (from  $d/dx$ ). However, the right-hand side of the equation has units of  $x^2$  (the product of  $ax$ ).
- Consequently, something is wrong as the equation is not dimensionally consistent.



### Questions

- Which of the following best represents the force needed to lift a heavy suitcase?  
a. 25 N                      b. 25 kN                      c. 250 N                      d. 250 kN
- Pick the correct answer(s); a watt is  
a. one joule per second    b. equal to  $1 \text{ (kg)(m}^2\text{)/s}^2$     c. the unit for all types of power  
d. all of the above                      e. none of the above
- Is kg/s a basic or derived unit in SI?
- Answer the following questions yes or no. Can you  
a. divide ft by s?   b. divide m by cm?   c. multiply ft by s?   d. divide ft by cm?   e. divide m by (deg) K?   f. add ft and s?   g. subtract m and (deg) K   h. add cm and ft?   i. add cm and  $\text{m}^2$ ?  
j. add 1 and 2 cm?
- Why is it not possible to add 1 ft and  $1 \text{ ft}^2$ ?
- What is  $g_c$ ?
- Is the ratio of the numerator and denominator in a conversion factor equal to unity?
- What is the difference, if any, between pound force and pound mass in the AE system?
- Could a unit of force in the SI system be kilogram force?
- Contrast the procedure for converting units within the SI system with that for the AE system.
- What is the weight of a one pound mass at sea level? Would the mass be the same at the center of Earth? Would the weight be the same at the center of Earth?
- What is the mass of an object that weighs 9.80 kN at sea level?
- Explain what dimensional consistency means in an equation.
- Explain why the so-called dimensionless group has no net dimensions.
- If you divide all of a series of terms in an equation by one of the terms, will the resulting series of terms be dimensionless?
- How might you make the following variables dimensionless:  
a. Length (of a pipe).    b. Time (to empty a tank full of water).

### Answers:

- (c)
- (a)
- Derived.
- (a) - (e) yes; (f) and (g) no; (h) and (i) no; (j) no.



5. The dimensions are not the same.
6. A conversion factor in the American Engineering system of units.
7. Yes.
8.  $\text{lb}_f$  is force and  $\text{lb}_m$  is mass, and the dimensions are different.
9. The unit is not legal in SI.
10. In SI the magnitudes of many of the units are scaled on the basis of 10, in AE.  
Consequently, the units are often ignored in making conversion in SI.
11. (a) 1  $\text{lb}_f$  in the AE system of units; (b) yes; (c) no.
12. 1000 kg.
13. All additive terms on the right-hand side of an equation must have the same dimensions as those on the left-hand side.
14. All of the units cancel out.
15. Yes.
16. (a) Divide by the radius or diameter; (b) divide by the total time to empty the tank, or by a fixed unit of time.

### **Problems**

1. Classify the following units as correct or incorrect units in the SI system:  
a. nm      b. K      c. sec      d. N/mm      e.  $\text{kJ}/(\text{s})(\text{m}^3)$
2. Add 1 cm and 1 m.
3. Subtract 3 ft from 4 yards.
4. Divide  $3 \text{ m}^{1.5}$  by  $2 \text{ m}^{0.5}$ .
5. Multiply 2 ft by 4 lb.
6. What are the value and units of  $g_c$  in the SI system?
7. Electronic communication via radio travels at approximately the speed of light (186,000 miles/second). The edge of the solar system is roughly at Pluto, which is  $3.6 \times 10^9$  miles from Earth at its closest approach. How many hours does it take for a radio signal from Earth to reach Pluto?
8. Determine the kinetic energy of one pound of fluid moving in a pipe at the speed of 3 feet per second.
9. Convert the following from AE to SI units:  
a.  $4 \text{ lb}_m/\text{ft}$  to  $\text{kg}/\text{m}$       b.  $1.00 \text{ lb}_m/(\text{ft}^3)(\text{s})$  to  $\text{kg}/(\text{m}^3)(\text{s})$
10. Convert the following  $1.57 \times 10^{-2} \text{ g}/(\text{cm})(\text{s})$  to  $\text{lb}_m/(\text{ft})(\text{s})$
11. Convert 1.1 gal to  $\text{ft}^3$ .



12. Convert 1.1 gal to m<sup>3</sup>.

13. An orifice meter is used to measure the rate of flow of a fluid in pipes. The flow rate is related to the pressure drop by the following equation

$$u = c \sqrt{\frac{\Delta P}{\rho}}$$

Where  $u$  = fluid velocity

$\Delta p$  = pressure drop 1force per unit area<sup>2</sup>

$\rho$  = density of the flowing fluid

$c$  = constant

What are the units of  $c$  in the SI system of units?

14. The thermal conductivity  $k$  of a liquid metal is predicted via the empirical equation

$$k = A \exp (B/T)$$

where  $k$  is in J/(s)(m)(K) and  $A$  and  $B$  are constants. What are the units of  $A$  and  $B$ ?

### Answers:

1. (a), (d), (e) are correct.
2. Change units to get 101 cm.
3. Change units to get 9 ft.
4. 1.5 m.
5. 8 (ft)(lb).
6. 1, dimensionless.
7. 5.38 hr.
8. 0.14 (ft) (lb<sub>f</sub>).
9. a. 5.96 kg/m; b. 16.0 kg/(m<sup>3</sup>)(s)
10.  $1.06 * 10^{-3}$  lb<sub>m</sub>/(ft)(s)
11. 0.15 ft<sup>3</sup>
12.  $4.16 * 10^{-3}$  m<sup>3</sup>.
13.  $c$  is dimensionless
14.  $A$  has the same units as  $k$ ;  $B$  has the units of T



**Supplementary Problems (Chapter One):**

**Problem 1**

Convert the following quantities to the ones designated :

- 42 ft<sup>2</sup>/hr to cm<sup>2</sup>/s.
- 25 psig to psia.
- 100 Btu to hp-hr.

**Solution**

$$\text{a. } \frac{42.0 \text{ ft}^2}{\text{hr}} \left| \frac{1.0 \text{ m}}{3.2808 \text{ ft}} \right|^2 \left| \frac{10^4 \text{ cm}^2}{1.0 \text{ m}^2} \right| \left| \frac{1 \text{ hr}}{3600 \text{ s}} \right| = 10.8 \text{ cm}^2/\text{s}$$

$$\text{b. } \frac{100 \text{ Btu}}{1 \text{ Btu}} \left| \frac{3.93 \times 10^{-4} \text{ hp-hr}}{1 \text{ Btu}} \right| = 3.93 \times 10^{-2} \text{ hp-hr}$$

$$\text{c. } \frac{80.0 \text{ lb}_f}{(\text{lb}_f)(\text{s})^2} \left| \frac{32.174 (\text{lb}_m)(\text{ft})}{2.20 \text{ lb}_m} \right| \left| \frac{1 \text{ kg}}{2.20 \text{ lb}_m} \right| \left| \frac{1 \text{ m}}{3.2808 \text{ ft}} \right| \left| \frac{1 \text{ N}}{1 (\text{kg})(\text{m})(\text{s})^{-2}} \right| = 356 \text{ N}$$

**Problem 2**

Convert the ideal gas constant :  $R = 1.987 \frac{\text{cal}}{(\text{gmol})(\text{K})}$  to  $\frac{\text{Btu}}{(\text{lb mol})(^\circ\text{R})}$

**Solution**

$$\frac{1.987 \text{ cal}}{(\text{gmol})(\text{K})} \left| \frac{1 \text{ Btu}}{252 \text{ cal}} \right| \left| \frac{454 \text{ gmol}}{1 \text{ lb mol}} \right| \left| \frac{1 \text{ K}}{1.8 ^\circ\text{R}} \right| = 1.98 \frac{\text{Btu}}{(\text{lb mol})(^\circ\text{R})}$$

**Problem 3**

Mass flow through a sonic nozzle is a function of gas pressure and temperature. For a given pressure  $p$  and temperature  $T$ , mass flow rate through the nozzle is given by

$$m = 0.0549 p / (T)^{0.5} \quad \text{where } m \text{ is in lb/min, } p \text{ is in psia and } T \text{ is in } ^\circ\text{R}$$

- Determine what the units for the constant 0.0549 are.
- What will be the new value of the constant, now given as 0.0549, if the variables in the equation are to be substituted with SI units and  $m$  is calculated in SI units.

**Solution**

- Calculation of the constant.



The first step is to substitute known units into the equation.

$$\frac{\text{lb}_m}{\text{min}} = 0.0549 \frac{\text{lb}_f}{(\text{in}^2)(^\circ\text{R})^{0.5}}$$

$$\frac{\text{lb}_f}{(\text{in}^2)(^\circ\text{R})^{0.5}} \left| \frac{(\text{lb}_m)(\text{in})^2(^\circ\text{R})^{0.5}}{(\text{min})(\text{lb}_f)} \right| \text{-----} \rightarrow \frac{(\text{lb}_m)}{(\text{min})} \frac{(\text{lb}_m)(\text{in})^2(^\circ\text{R})^{0.5}}{(\text{min})(\text{lb}_f)}$$

Units for the constant 0.0549 are

- b. To determine the new value of the constant, we need to change the units of the constant to appropriate SI units using conversion factors.

$$\frac{0.0549 (\text{lb}_m)(\text{in}^2)(^\circ\text{R})^{0.5}}{(\text{lb}_f)(\text{min})} \left| \frac{(0.454 \text{ kf})}{(1 \text{ lb}_m)} \right| \left| \frac{(14.7 \text{ lb}_f / \text{in}^2)}{101.3 \times 10^3 \text{ N/m}^2} \right| \left| \frac{(1 \text{ min})}{(60 \text{ s})} \right| \left| \frac{(1 \text{ K})^{0.5}}{(1.8 ^\circ\text{R})^{0.5}} \right| \left| \frac{(\text{p})}{(\text{T})^{0.5}} \right|$$

$$\text{m} = 4.49 \times 10^{-8} (\text{m}) (\text{s}) (\text{K})^{0.5} \frac{(\text{p})}{(\text{T})^{0.5}}$$

Substituting pressure and temperature in SI units

$$\text{m} = 4.49 \times 10^{-8} (\text{m}) (\text{s}) (\text{K})^{0.5} \frac{(\text{p})(\text{N/m}^2)}{(\text{T})^{0.5}(\text{K})^{0.5}} \left| \frac{1 \text{ kg}/(\text{m})(\text{s})^2}{1 \text{ N/m}^2} \right|$$

$$\text{m} \frac{(\text{kg})}{(\text{s})} = 4.49 \times 10^{-8} \frac{(\text{p})}{(\text{T})^{0.5}} \quad \text{where p is in N/m}^2 \text{ and T is in K}$$

#### Problem 4

An empirical equation for calculating the inside heat transfer coefficient,  $h_i$ , for the turbulent flow of liquids in a pipe is given by:

$$h_i = \frac{0.023 G^{0.8} K^{0.67} C_p^{0.33}}{D^{0.2} \mu^{0.47}}$$

where  $h_i$  = heat transfer coefficient, Btu/(hr)(ft)<sup>2</sup>(°F)

$G$  = mass velocity of the liquid, lb<sub>m</sub>/(hr)(ft)<sup>2</sup>

$K$  = thermal conductivity of the liquid, Btu/(hr)(ft)(°F)

$C_p$  = heat capacity of the liquid, Btu/(lb<sub>m</sub>)(°F)

$\mu$  = Viscosity of the liquid, lb<sub>m</sub>/(ft)(hr)

$D$  = inside diameter of the pipe, (ft)

- Verify if the equation is dimensionally consistent.
- What will be the value of the constant, given as 0.023, if all the variables in the equation are inserted in SI units and  $h_i$  is in SI units.



### Solution

- a. First we introduce American engineering units into the equation:

$$h_i = \frac{0.023 \left[ (\text{lb}_m) / (\text{ft})^2 (\text{hr}) \right]^{0.80} \left[ \text{Btu} / (\text{hr})(\text{ft})(^\circ\text{F}) \right]^{0.67} \left[ \text{Btu} / (\text{lb}_m)(^\circ\text{F}) \right]^{0.33}}{(\text{ft})^{0.2} \left[ \text{lb}_m / (\text{ft})(\text{hr}) \right]^{0.47}}$$

$$h_i = \frac{0.023(\text{Btu})^{0.67} (\text{lb}_m)^{0.8}}{[(\text{lb}_m)^{0.33}(\text{lb}_m)^{0.47}]} \left| \frac{(\text{ft})^{0.47}}{[(\text{ft})^{1.6}(\text{ft})^{0.67}(\text{ft})^{0.2}]} \right| \left| \frac{(1)}{[(^\circ\text{F})^{0.67}(\text{ft})^{0.33}]} \right| \left| \frac{(\text{hr})^{0.47}}{[(\text{hr})^{0.8}(\text{hr})^{0.67}]} \right|$$

$$h_i = 0.023 \frac{\text{Btu}}{(\text{hr})(\text{ft})^2 (^\circ\text{F})}$$

The equation is dimensionally consistent.

- b. The constant 0.023 is dimensionless; a change in units of the equation parameters will not have any effect on the value of this constant.